Author(s): Bossuyt, Sven

Title: Optimized patterns for digital image correlation

Year: 2013

Version: Post print

Please cite the original version:

Bossuyt, Sven. 2013. Optimized patterns for digital image correlation. Proceedings of the 2012 Annual Conference on Experimental and Applied Mechanics, Volume 3: Imaging Methods for Novel Materials and Challenging Applications,. ISBN 978-1-4614-4235-6 (electronic). ISBN 978-1-4614-4234-9 (printed).

Optimized patterns for digital image correlation

Sven Bossuyt*
* Aalto University, Department of Engineering Design and Production,
P.O. Box 14200, FI-00076 Helsinki, Finland

Abstract

This work presents theoretical background on a novel class of strain sensor patterns. A combination of morphological image processing and Fourier analysis is used to characterize gray-scale images, according to specific criteria, and to synthesize patterns that score particularly well on these criteria. The criteria are designed to evaluate, with a single digital image of a pattern, the suitability of a series of images of that pattern for full-field displacement measurements by digital image correlation (DIC). Firstly, morphological operations are used to flag large featureless areas and to remove from consideration features too small to be resolved. Secondly, the autocorrelation peak sharpness radius en the autocorrelation margin are introduced to quantify the sensitivity and robustness, respectively, expected when using these images in DIC algorithms. For simple patterns these characteristics vary in direct proportion to each other, but it is shown how to synthesize a range of patterns with wide autocorrelation margins even though the autocorrelation peaks are sharp. Such patterns are exceptionally well-suited for DIC measurements.

1 Introduction

Techniques using pattern matching in digital images, commonly referred to as Digital Image Correlation (DIC), are now widely used for measuring shape, motion and deformation[1]. Quantitative error analysis[2, 3, 4, 5] of these techniques shows that the measurement error depends critically on the presence of large intensity gradients in the image. Therefore, obtaining satisfactory results with the technique often requires a sample preparation procedure to enhance the image texture of the sample. Various methods for generating suitable textures at microscopic length scales[6, 7, 8] have been reported in detail. At macroscopic length scales, a suitable texture can often be achieved by spraying paint speckles onto the object.

In addition to methods for patterning objects for the purpose of DIC measurements, methods for characterizing the resulting patterns[9, 10, 11] and for choosing the analysis parameters in function of the characteristics of the pattern[12, 13] or in function of the strain field[14] have been proposed. However, questions remain about the reliability of DIC measurements, in terms of spatial resolution, accuracy, precision, sensitivity, and robustness of the measurement. It is well established that there is a trade-off between spatial resolution and precision, which governs the choice of subset size when analyzing DIC measurements, and which is affected by the quality of the pattern.

2 Strain sensor patterns

2.1 Displacement sensitivity

The sensitivity of a DIC measurement is the smallest displacement that causes a statistically significant change in the image of the strain sensor pattern. It is closely related to the precision of the displacement measurement, which is the smallest change in displacement that causes a statistically significant change in the image. In particular, since changes in the image are quantified by the cross-correlation with the original image –or by some similar measure that is practically equivalent for this purpose— it is the rate of change of the correlation with small displacements, compared to the statistical variation in the correlation arising from measurement errors, that determines the displacement precision and sensitivity of DIC measurements. The variability of the correlation is strongly affected by experimental conditions and by analysis parameters such as the size of the image subset used for calculating the correlation, but these effects are nearly identical for DIC measurements with different strain sensor patterns and otherwise identical conditions. It is in the rate of change of the correlation with small displacements, i.e., in the sharpness of the correlation peak, that the effect of the strain sensor pattern on the sensitivity of the DIC measurement manifests itself. High-contrast patterns with many small features result in a sharp correlation peak, and impart high sensitivity on DIC measurements.

Features too small to be resolved by the imaging units, however, should be excluded from the argument above. Small features contribute disproportionately to the sharpness of the correlation of an image with itself. In the cross-correlations of different images of the pattern, however, because too-small features are inaccurately transformed by interpolation back to

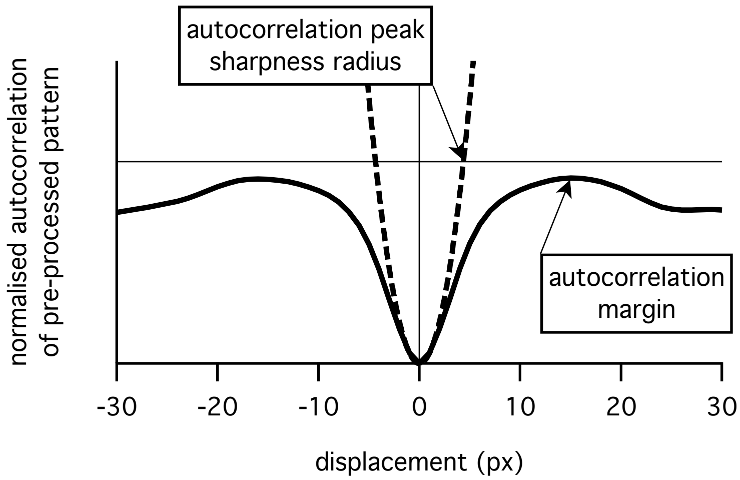

Figure 1: 1-dimensional section through an autocorrelation peak, showing the parabolic extrapolation of the mean value of the autocorrelation for displacements of 1 pixel, to obtain the autocorrelation peak sharpness radius, and the position of the rim of the peak used to determine the autocorrelation margin.

the undeformed state where the correlation is calculated, too-small features contribute primarily to the noise. Nevertheless, it is desirable for a criterion quantifying a strain sensor pattern's suitability —in this case with respect to sensitivity— for DIC measurements, to be evaluated on the basis of the pattern itself or a single image of the pattern, rather than a series of images from which cross-correlations could be calculated. A low-pass convolution filter could be used to remove too-small features from the image, but this has the disadvantage of also attenuating sharp edges of features large enough to be resolved. Morphological operations on images do not have this disadvantage. In particular, gray-scale opening and closing operations will remove small bright and dark features from the image, without reducing the contrast of sharp features that are larger than the operations' kernel. The combination of morphological opening and closing may be referred to as morphological smoothing. In accordance with the general rule of thumb for DIC that the feature size should be at least 3 pixels, morphological smoothing with a 3×3 kernel is proposed for pre-processing the image of the pattern, after which the autocorrelation of the pre-processed pattern can be used as a proxy for the cross-correlations of different images of the pattern.

The autocorrelation peak sharpness radius of a pre-processed image of the pattern is proposed to quantitatively evaluate how a particular strain sensor pattern influences the sensitivity of a DIC measurement. It is calculated as illustrated in figure 1, from the autocorrelation value for zero displacement, $A|_0$, and the mean of the autocorrelation values for displacements by 1 pixel along the 4 cardinal directions of the pixel grid, $\langle A|_{\pm 1} \rangle$, by extrapolating a parabolic peak shape to the autocorrelation value A_∞ for full-contrast random noise[1]:

$$R_{\text{peak}} = \sqrt{\frac{A_\infty - A|_0}{\langle A|_{\pm 1} \rangle - A|_0}}. \tag{1}$$

Patterns with more features, higher contrast and sharper edges have a smaller autocorrelation peak sharpness radius, corresponding to better displacement sensitivity in DIC measurements. For anisotropic patterns, in agreement with the work of Triconnet et al.[12], the extrapolation should use an elliptic paraboloid instead of a paraboloid of revolution, replacing the peak sharpness radius by the orientation and two semi-axes of an ellipse.

2.2 Spatial resolution

The DIC technique allows full-field displacement measurements, i.e., it theoretically gives values for the displacement at every point within the field of view. In practice, each value of the displacement is associated not so strictly with an idealized point in the field of view, but with a small domain surrounding that point. When these domains for two displacement measurements overlap, then those measurements are not independent of each other. To improve the spatial resolution of the DIC measurement, the displacements should be determined using information from smaller domains. This tendency is opposed by the need for displacement precision and sensitivity of the DIC measurement, which improves when the domains are larger. Thus, there is an opportunity in the analysis of the raw measurement data to consider a trade-off between displacement precision and spatial resolution of the results.

Nevertheless, there are practical limits to this trade-off. If there is too much variation in the displacement within the domain, then the information content from a smaller domain gains in quality what it loses in quantity: the domains should be smaller

[1] In DIC, the "correlation" is often defined so that it is 0 for identical images, rather than for uncorrelated images.

than the length scale of the variations in the displacement field to be measured. This is not merely an observation that the measurement would fail to represent small details in the displacement field: if none of the hypothetical displacement fields considered can provide a good correlation between predicted and observed digital images, then the DIC measurement fails entirely. Conversely, if there is not enough variation in the image within the domain, then the DIC technique can not directly give information about the displacement at that point: the domains should be larger than featureless areas in the image. It follows that the pattern must have features small compared to the length scale of the variations in the displacement field, but large enough to be resolved in the image, for DIC to be useful. Furthermore, the largest featureless areas should be not much larger than the smallest features. The spatial resolution of the DIC measurement is then practically limited by aspects of the actual displacement field to be measured and of the pattern to be imaged.

As a criterion for the suitability of a pattern for DIC measurements, it is the size of the largest featureless areas that determines the spatial resolution that can be achieved in DIC measurements using that pattern. Morphological operations with large structuring elements will expand and merge all features from an image, except those where a featureless area larger than the structuring element occurs. They can therefore be used to detect featureless areas of a given size. The concept of morphological granulometry[15], using repeated morphological operations at different sizes, is used to characterize the size distribution of features in an image. In the special case where only round features in a binary image are considered, fast algorithms for the Euclidean distance transform provide efficient implementations of morphological granulometry and morphological operations with large structuring elements.

2.3 Robustness

It is always possible that the DIC algorithm finds a local optimum in the correlation, at a displacement that differs from the true displacement. In general, there are many such local optima. The result of the DIC measurement is completely wrong, when the algorithm doesn't end at the correct optimum. The accuracy of the DIC measurement therefore depends on the robustness of the optimization strategy.

If the image is not self-similar and the hypothetical image that the DIC algorithm would predict for the true displacement field is a good approximation of the observed image, then the local optimum in the correlation at the true displacement coincides with the global optimum. This is not to say that carrying out a global optimization algorithm every time, for each displacement value in the full displacement field, multiplied by the number of frames in an image series, would be practical. Local optimization algorithms, especially when provided with good initial guesses, are much more efficient computationally. Rather, if it is known that the correlation passes a certain threshold if and only if the displacement is close to the true displacement, then the robustness of the optimization strategy can be improved by rejecting any results that do not meet that threshold, or reevaluating those results.

In addition to cross-checks allowing to reject "false positive" matches, it is important to provide the DIC algorithm with good initial guesses for the displacement. How good those initial guesses for the displacement need to be, depends on the pattern, more specifically on the pattern's correlation peak. Local optimization algorithms use local information, such as the gradient of the correlation, to determine a promising direction for improving the correlation, and then search in that direction. Close enough to a local optimum, the gradient of the correlation always points toward that local optimum. If the concept correlation peak is interpreted more precisely as the range of displacements —and the associated correlation values— from which the gradient loads to a particular local optimum in the correlation, then the requirement for the initial guess is that it should be within the main correlation peak, i.e., the correlation peak corresponding to the true displacement. In image processing, by analogy with the flow of water downhill in a mountain landscape, the name watershed segmentation is used for an algorithm for associating initial positions with the local minimum that is reached from that position by going (downward) along the direction of the gradient. Thus the watershed segmentation allows to characterize for a pattern how that pattern's correlation peak affects the robustness of a DIC measurement.

For robust DIC measurements, the main correlation peak should therefore be broad enough to enclose the margin of error on the initial guess for the displacement. Patterns consisting of large, smooth features give broad correlation peaks, but result in poor displacement sensitivity and poor spatial resolution for the DIC measurement. The shape of the correlation peak that would be ideal for DIC measurements superimposes a sharp tip —arising from a high density of small, sharp features in the pattern— onto a broad base —corresponding to a larger-scale modulation of the pattern— in such proportion that neither is obscured by the other or by the noise in the measured images. As a measure of the typical width at the rim of the main autocorrelation peak of a pre-processed image of a strain sensor pattern, in order to evaluate how that pattern influences the robustness of the DIC measurement, the autocorrelation margin is proposed: it is the radius of a circle with the same area as the range of displacements corresponding to the main correlation peak.

2.4 Reproducibility

In order to gain further acceptance as a reliable measurement method, DIC must be seen to be reproducible. Variability in the application of patterns when preparing objects for DIC measurements is of particular concern. This variability can be quantified by repeating the measurements proposed here —i.e., the autocorrelation peak sharpness radius, the autocorrelation margin, and the largest featureless areas, on images pre-processed by morphological opening and closing with a 3×3 kernel— on different

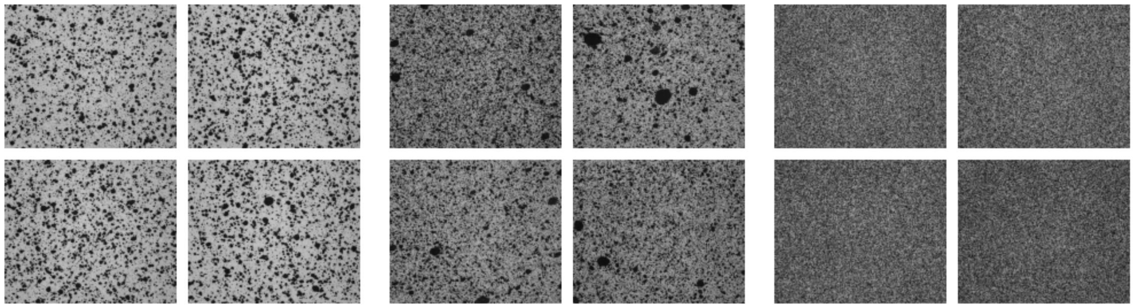

Figure 2: Variability of patterns applied by spray painting.

instances of a pattern. Especially with spray-painted patterns, the quality of the pattern is often inconsistent across different areas on the same object or across different objects, and highly dependent on the skill of the human operator, as illustrated in figure 2. It appears that DIC measurements obtained with random patterns applied by experienced users of DIC are often satisfactory, if the imaging conditions and analysis parameters are adapted according to the pattern. Nevertheless, it would be preferable to adapt the pattern to the measurement, rather than vice versa, and DIC measurements will be more reproducible when the judgement of experienced users and their dexterity in spraying paint can be replaced by algorithms and printers run by computers, to apply patterns optimized for each particular measurement.

3 How to generate optimized patterns

Methods closely related to the methods for calculating the proposed DIC suitability criteria can be used to generate patterns that score well on those criteria while satisfying practical constraints. In particular, Fourier transforms provide numerical methods that are computationally efficient for calculating correlation functions and also an abstract formalism that is convenient for reasoning about correlation functions. A useful corollary to the Fourier convolution theorem states that the Fourier transform of the autocorrelation of a pattern is the magnitude squared of the Fourier transform of that pattern. Therefore, specifying the amplitude of the Fourier transform, regardless of its phase, specifies the pattern's autocorrelation function. Conversely, a pattern is fully specified by its autocorrelation function and the phase of its Fourier transform. However, not just any function can be specified as the autocorrelation function: an autocorrelation function's Fourier transform must be positive. So for the purpose of designing patterns with certain characteristics of their autocorrelation function, it is convenient to start with the Fourier transform of the autocorrelation function. For isotropic patterns, the two-dimensional Fourier transform reduces to the Fourier-Bessel transform of the radial part, also known as the Hankel transform.

3.1 Ideally sharp patterns

Sharp edges or peaks require high frequencies in the Fourier transform. Since the phase of the Fourier transform of the pattern's autocorrelation function is identically zero, all frequencies interfere constructively at the main autocorrelation peak, so the autocorrelation peak of an isotropic band-limited pattern becomes sharper if and only if more of the pattern's spectrum is closer to the band limit. The sharpest possible autocorrelation peak is therefore obtained when all of the spectral content is right at the band limit. Figure 3 illustrates the appearance of patterns obtained by specifying a 2-D Fourier transform that within a narrow ring has unit amplitude and random phase, and is zero everywhere else. The autocorrelation function of such patterns is purely radial: it is the Hankel transform of a delta function, which works out to be the zeroth order Bessel function of the first kind. By construction, these patterns are isotropic and band-limited. The sizes of their features are extremely uniform, without variation of the average gray level at longer length scales.

A notable feature of these ideally sharp, isotropic, band-limited patterns' autocorrelation functions is that these patterns are clearly anti-correlated for displacements of approximately one feature diameter. For displacements larger than that, the autocorrelation oscillates. Thus the main autocorrelation peak is exceptionally well-defined, since the correlation difference from the peak to the margin is about 40% greater than with uncorrelated patterns. But since the correlation for displacements slightly greater than one feature diameter improves with increasing error, these patterns are also especially unforgiving of errors in the initial guesses for the displacements, when those errors exceed one feature diameter.

3.2 Gaussian speckle patterns

Figure 4 illustrates the appearance of patterns obtained by specifying a 2-D Gaussian for the Fourier transform amplitude, with random phase. This is equivalent to applying a Gaussian filter to white noise, since the Fourier transform of a Gaussian is again a Gaussian. The result is a typical speckle pattern, not quite as uniform as the ideally sharp pattern of figure 3, with smooth variations of the average gray level at all length scales larger than the typical feature size.

The autocorrelation of a Gaussian speckle pattern exhibits a simple peak, monotonically approaching the correlation value for uncorrelated patterns. As a consequence of the variations of the average gray level in these patterns, less of the gray scale range of the image is available as contrast of the features themselves. On the other hand, to make a pattern forgiving of errors in the initial guesses for the displacements, significant variations of the average gray level at length scales on the order of that displacement error are required.

3.3 Combined patterns

The ideal pattern for DIC would combine a sharp autocorrelation peak with a well-defined autocorrelation margin that is sufficiently broad to enclose the margin of error on the initial guesses for the displacement. If the autocorrelation margin is already sufficient with an ideally sharp pattern with features as small as can be reliably resolved by the imaging system, then the ideally sharp pattern is optimal. Otherwise, the resolution of the imaging system and the accuracy of the initial guesses define two distinct length scales for the optimization of the pattern. Figure 5 illustrates the pattern obtained by superimposing two ideally sharp patterns with two different length scales. The two length scales are clearly visible in the pattern, and the autocorrelation has nearly the peak sharpness of that of the finer pattern. However, this pattern also has approximately the same autocorrelation margin as the finer pattern, due to the oscillations in the autocorrelation of the ideally sharp pattern.

Figure 6 illustrates the pattern obtained by cutting off the low-frequency components of a Gaussian speckle pattern, and superimposing an ideally sharp pattern with that same cut-off frequency. This pattern exhibits the desired combination of a sharp autocorrelation peak with a broad, well-defined autocorrelation margin. By choosing the relative amplitude of the two superimposed patterns, the fraction of the gray scale range of the image devoted to peak sharpness, via the fine Gaussian speckle pattern, and to the autocorrelation margin, via the broad ideally sharp pattern, can be selected at will. The Gaussian speckle pattern can be made as fine as can still be resolved by the imaging system to be used in the experiment, and for any given accuracy of the initial guesses for the displacement, the feature size of the ideally sharp pattern can be chosen so that the autocorrelation margin of the combined pattern is sufficient.

3.4 Optimized patterns

The combined patterns using a Gaussian speckle pattern superimposed on an ideally sharp base component are, in some sense, already optimized for DIC, since they allow to tailor the autocorrelation of the pattern to the requirements of a DIC experiment. However, the constraints under which they were optimized are perhaps not the most relevant in practice.

In particular, constraining the patterns to be band-limited prevents the patterns from having high-contrast edges, even though increasing contrast is the most direct way of increasing the intensity gradients in the image, which in turn reduces many of the most important error terms in DIC measurements[2]. It is straightforward to increase the contrast of grayscale patterns, by choosing the new gray scale values as a function of the old gray scale values: where the slope of this function is greater than 1, contrast is increased. If the maximum possible gray scale range is already used, then it is necessary in order to increase contrast in some parts of the image, to reduce contrast in other parts of the image. By reducing contrast at gray scale levels that are not used much in the image, to increase contrast at the gray scale levels that are more used, overall contrast of the image increases while the histogram becomes more uniform. Choosing the inverse function of the histogram of the original gray scale values, as the function to transform those original gray scale values to the new gray scale values, results in a higher-contrast pattern where the histogram of the new gray scale values is completely equalized. Still higher contrast is reached by stretching the gray scale

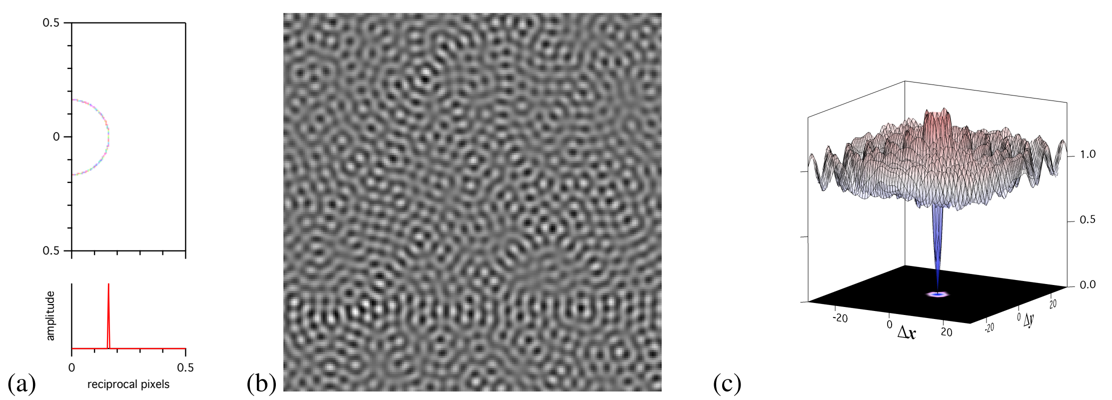

Figure 3: Ideally sharp, isotropic, band-limited pattern: (a) frequency content, (b) grayscale pattern, (c) surface plot of its 2-D autocorrelation

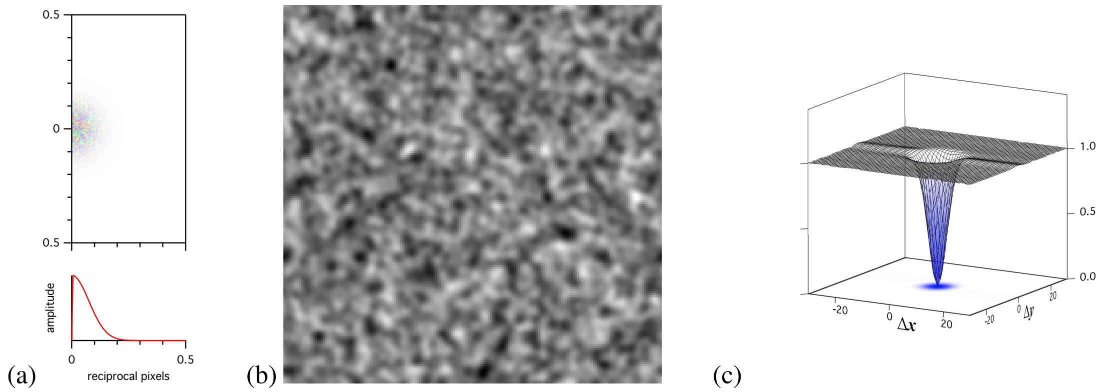

Figure 4: Speckle pattern with a Guassian spectrum: (a) frequency content, (b) grayscale pattern, (c) surface plot of its 2-D autocorrelation

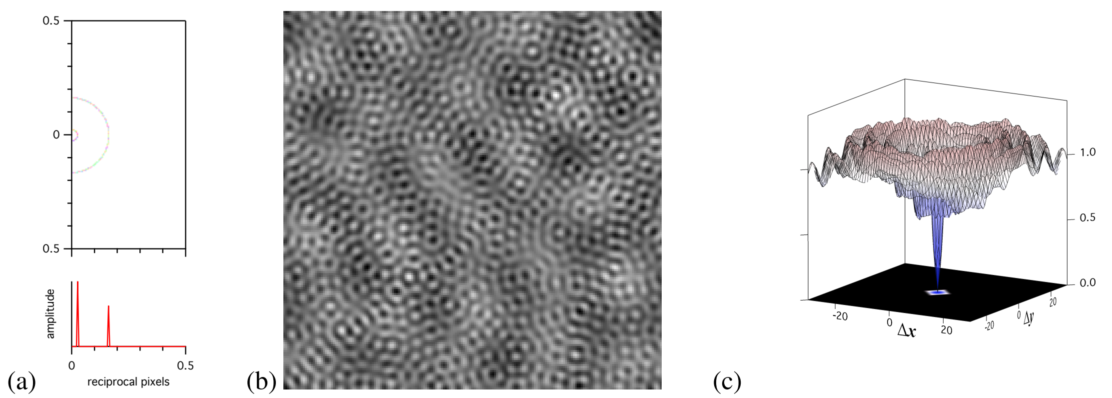

Figure 5: superposition of two ideally sharp patterns at different length scales: (a) frequency content, (b) grayscale pattern, (c) surface plot of its 2-D autocorrelation

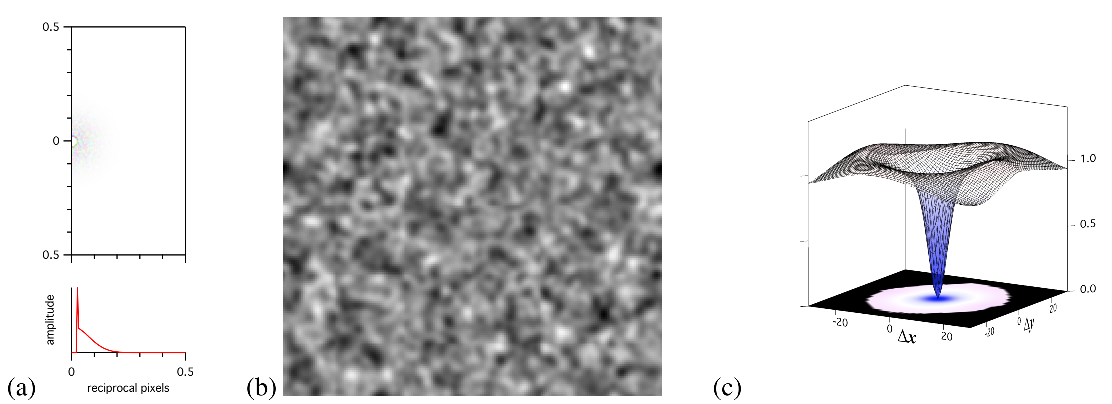

Figure 6: superposition of an ideally sharp base component and a Gaussian speckle pattern: (a) frequency content, (b) grayscale pattern, (c) surface plot of its 2-D autocorrelation

beyond the maximum possible gray scale range in the image, and clipping the values that extend past that range to the extrema of the range, but that introduces sharp edges in the pattern. In the most extreme case, a black and white pattern remains and the operation is equivalent to thresholding the gray scale pattern.

Figure 7 illustrates that one of the effects of increasing contrast is indeed to improve the autocorrelation peak sharpness. As non-linear operations, the different methods of increasing contrast have complicated effects on the overall shape of the pattern's autocorrelation, which can be regarded as the addition of noise to the autocorrelation. If the base component of the combined pattern is sufficiently pronounced, then even the most extreme contrast increase does not significantly deteriorate the autocorrelation margin. Extreme contrast increases do cause problems with respect to two other pattern characteristics desired for DIC measurements: features can become very small if the threshold is close to a local extremum of the gray scale level, or disappear entirely —leaving possibly large featureless areas— when the threshold crosses the extremum.

To optimize the patterns for DIC, these problems should be corrected. Too-small features are easily removed by a morphological smoothing operation, but that exacerbates the appearance of large featureless areas. The following scheme is implemented to reintroduce features in the featureless areas. First, the signed Euclidean distance to the level set defined by the threshold value is calculated. The maximum allowed featureless area is subtracted from the absolute value of that distance, leaving a signed excess distance to the nearest feature. Scaling this function to a gray scale range that is small but not negligible compared to the gray scale range of the base component of the pattern gives a threshold adjustment term that brings out the speckle component of the pattern again, in featureless areas. However, to avoid disturbing the intended characteristics of the pattern too much, it is beneficial to not use this adjustment term directly. Applying a band-pass filter to the adjustment term will prevent it from introducing sharp edges of its own into the pattern, and correct for changes in the average gray scale level[2] with opposing changes in nearby regions of the pattern. Taking into account that larger autocorrelation margins, and smaller ratios of minimum feature size to maximum featureless area, are more demanding requirements for the optimization of a pattern, the parameters chosen for the pattern should be reasonable. But for reasonable parameters, a few iterations of adjusting the threshold like this to repair large featureless areas, and each time removing too-small features by morphological smoothing, usually result in a pattern with all the characteristics desired to make the pattern ideally suited for a DIC experiment.

4 Conclusions

A systematic analysis of the requirements of DIC measurements leads directly to a number of characteristics of image textures that quantify different aspects of their suitability for DIC measurements. Pre-processing an image with morphological operations to remove small features, allows these characteristics to be evaluated from a single image. Patterns well-suited for DIC exhibit a sharp correlation peak, a broad correlation margin, and have neither features too small to be resolved by the imaging system to be used nor large featureless areas. For isotropic, band-limited grayscale patterns, the optimization of the first three characteristics can proceed exactly, allowing to create patterns tailored for a specific DIC experiment. If the requirement that the patterns should be band-limited is relaxed, the contrast of these patterns can be increased, which should make them even more suitable for DIC. In the extreme case of black-and-white patterns, and following an iterative procedure to remove too-small features and repair large featureless areas, a novel class of strain sensor patterns are created that are particularly well-suited for DIC measurements.

5 Acknowledgements

This research is funded by the ISMO project of the Multidisciplinary Institute for Digitalisation and Energy (MIDE) of Aalto University and by the Academy of Finland. Experimental verification of the performance of DIC measurements using patterns such as those described here has been carried out in collaboration with Bachir Belkassem at the Vrije Universiteit Brussel.

References

[1] Sutton, M. A., Orteu, J. J. & Schreier, H. W. *Image Correlation for Shape, Motion and Deformation Measurements: Basic Concepts, Theory and Applications* (Springer Verlag, 2009).

[2] Wang, Y. Q., Sutton, M. A., Bruck, H. A. & Schreier, H. W. Quantitative error assessment in pattern matching: Effects of intensity pattern noise, interpolation, strain and image contrast on motion measurements. *Strain* **45**, 160–178 (2009).

[3] Bornert, M. *et al.* Assessment of digital image correlation measurement errors: methodology and results. *Experimental mechanics* **49**, 353–370 (2009).

[4] Dupré, J.C., Bornert, M., Robert, L. & Wattrisse, B. Digital image correlation: displacement accuracy estimation. *EPJ Web of Conferences* **6**, 31006 (2010).

[2] For black-and-white patterns, the average gray scale level is to be interpreted as the average density of white pixels.

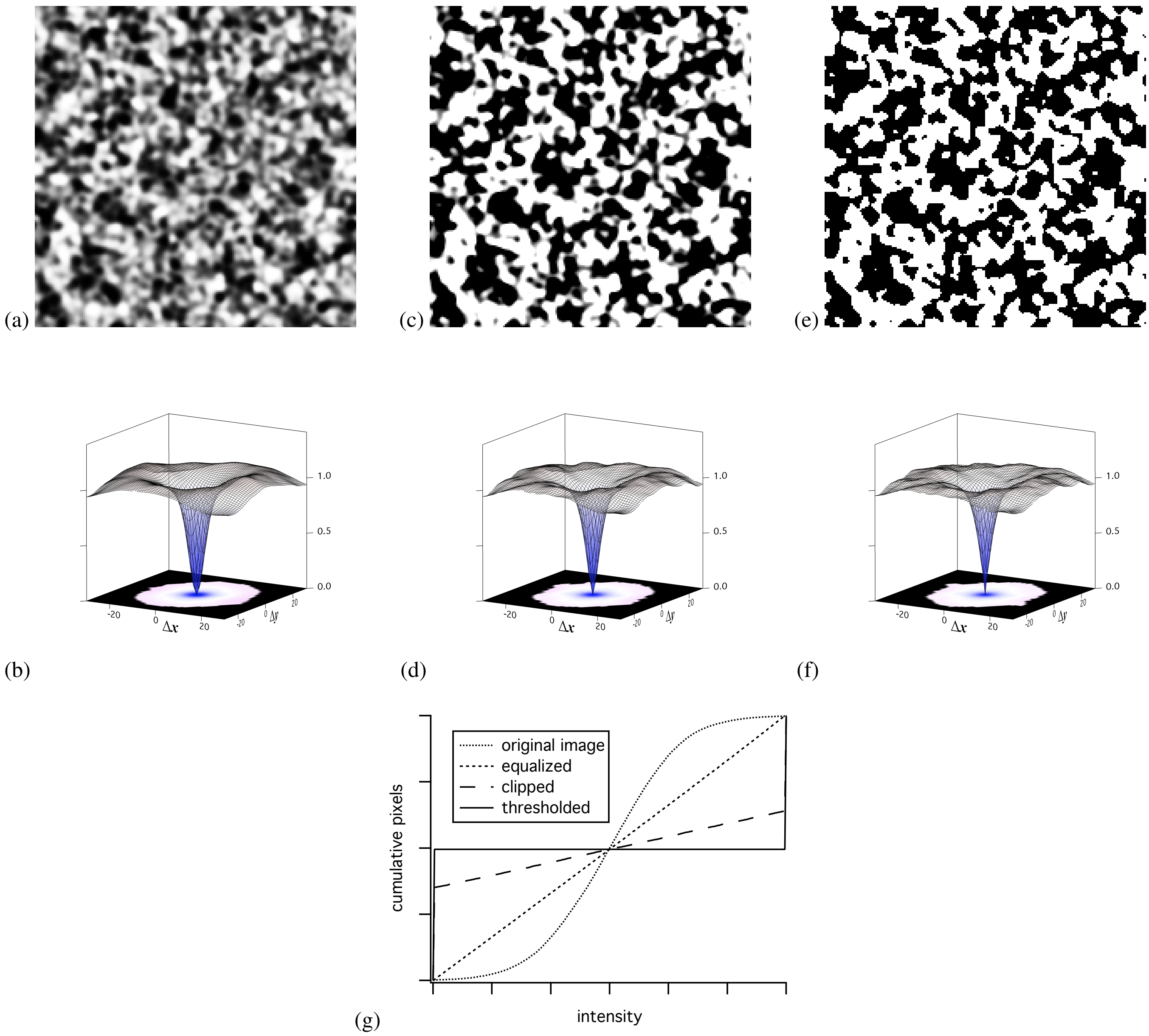

Figure 7: effect of increasing contrast on the pattern of Figure 6: (a) pattern after histogram equalization, with (b) its autocorrelation, (c) pattern after stretching contrast by a factor of 8, with (d) its autocorrelation, (e) pattern after thresholding at its mean gray level, with (f) the autocorrelation of that, and (g) the corresponding histograms

[5] Pan, B., Qian, K., Xie, H. & Asundi, A. On errors of digital image correlation due to speckle patterns. In He, X., Xie, H. & Kang, Y. (eds.) *ICEM 2008: International Conference on Experimental Mechanics 2008*, vol. 7375, 73754Z (SPIE, 2008).

[6] Collette, S. A. *et al.* Development of patterns for nanoscale strain measurements: I. fabrication of imprinted Au webs for polymeric materials. *Nanotechnology* **15**, 1812 (2004).

[7] Scrivens, W. A. *et al.* Development of patterns for digital image correlation measurements at reduced length scales. *Experimental Mechanics* **47**, 63–77 (2007).

[8] Kammers, A. D. & Daly, S. Small-scale patterning methods for digital image correlation under scanning electron microscopy. *Measurement Science and Technology* **22**, 125501 (2011).

[9] Lecompte, D. *et al.* Quality assessment of speckle patterns for digital image correlation. *Optics and lasers in Engineering* **44**, 1132–1145 (2006).

[10] Fazzini, M., Mistou, S., Dalverny, O. & Robert, L. Study of image characteristics on digital image correlation error assessment. *Optics and Lasers in Engineering* **48**, 335–339 (2010).

[11] Ning, J. *et al.* Speckle patterning of soft tissues for strain field measurement using digital image correlation: Preliminary quality assessment of patterns. *Microscopy and Microanalysis* **17**, 81–90 (2011).

[12] Triconnet, K., Derrien, K., Hild, F. & Baptiste, D. Parameter choice for optimized digital image correlation. *Optics and Lasers in Engineering* **47**, 728–737 (2009).

[13] Cofaru, C., Philips, W. & Paepegem, W. V. Improved newton–raphson digital image correlation method for full-field displacement and strain calculation. *Appl. Opt.* **49**, 6472–6484 (2010).

[14] Schreier, H. W. & Sutton, M. A. Systematic errors in digital image correlation due to undermatched subset shape functions. *Experimental Mechanics* **42**, 303–310 (2002).

[15] Weeks, A. R., Jr. *Fundamentals of electronic image processing*. SPIE/IEEE series on imaging science & engineering (IEEE Press, 1996).